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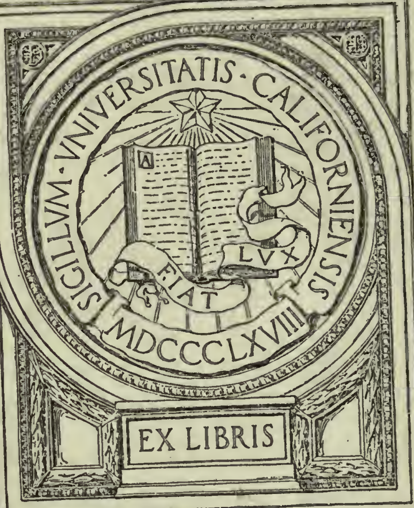

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4. The Minute Mathematician.

By Bishop Berkeley
and James Jurin

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ERRATA.

Page 16. l. 20. r. *contemnendi* *.
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2

GEOMETRY
No FRIEND to
INFIDELITY:
OR, A
DEFENCE
OF
Sir *ISAAC NEWTON*
AND THE
Britiſh Mathematicians,

In a LETTER to the Author of the *Analyſt*.

Wherein it is examined

How far the Conduct of ſuch Divines as intermix the Intereſt of Religion with their private Diſputes and Paſſions, and allow neither Learning nor Reaſon to thoſe they differ from, is of Honour or Service to Chriſtianity, or agreeable to the Example of our Bleſſed Saviour and his Apoſtles.

By PHILALETHES CANTABRIGIENSIS. ~~Dr Pemberton~~, ~~or~~ Dr. Jurin.

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s interſit, niſi dignus vindice nodus

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ERRATA.

Pag. 22. line 8. for *of* read *in*. p. 56. l. ult. dele *an*. p. 59. l. 15. r. *and me*. p. 60. l. 1, r. *him?*

A
LETTER
To the AUTHOR of the
ANALYST.

Cambridge, April 10. 1734.

SIR,

AS I am one of thoſe many perſons in this Univerſity, who have profited by your learned writings, and who greatly admire the depth of thought, the force of reaſon, and the perſpicuity of expreſſion that generally appears in them; I cannot but be extremely ſurprized and concerned, that a Gentleman of your abilities ſhould have taken ſo much pains not only to depreciate one of the nobleſt of all the ſciences, but to diſparage, to traduce, and even to defame a ſet of learned men, whoſe labours ſo greatly conduce to the honour of this iſland, and to the general good of mankind. You will eaſily ſee that I ſpeak of Mathematicks, and the *Britiſh* Mathematicians, and of the manner in which you have been pleaſed to treat them in your late pamphlet, intitled, *The Analyſt.* That diſcourſe, it is true, is ſeemingly directed to one perſon only; but you are not got through your firſt ſection, before you ſpeak of *too many more of the like character* with him; and in the contents to that very ſection, as well as in all the reſt of your diſcourſe, you talk of Mathematicians in ſo general a manner, that I am afraid your readers will think very few, if any, of thoſe Gentlemen are to be excepted out of that number, againſt which you bring ſo heavy a charge.

This charge, Sir, conſiſts of three principal points.

1. Of infidelity with regard to the Chriſtian Religion.
2. Of endeavouring to make others Infidels, and ſucceeding in thoſe endeavours by means of *the deference* which is paid *to their judgment, as being preſumed to be of all men the greateſt maſters of reaſon.*
3. Of error and falſe reaſoning in their own ſcience.

This, Sir, is the charge you have been pleaſed to bring againſt our Mathematicians, and I intend particularly to examine with what reaſon and truth you have done ſo. But before I do this, I muſt beg leave to make a few obſervations upon the intent and deſign of your treatiſe, the advantage to be expected from it, the prudence, and juſtice and honour of this deſign; and after that to inquire whether the motive to your writing it were really what you pretend and avow.

If your deſign were to be gueſſed at from your Title-page, wherein you profeſs to *examine, whether the object, principles and inferences of the modern Analyſis are more diſtinctly conceived, or more evidently deduced, than religious myſteries and points of faith*, one would be apt to conclude your intent was to ſhew, that the myſteries of Religion might be as clearly conceived, as the object and principles of the modern Analyſis; and that the ſeveral points of our Faith might be as evidently deduced, as the inferences of that Analyſis are from its principles; or, in other words, that you were about to give us a Mathematical demonſtration, or one of equal clearneſs and certainty, of the truth of the

Chriſtian Religion. This, I ſay, from the words of your Title-page one would naturally take to be your deſign: but it is ſo far from being ſo, that throughout your whole diſcourſe, though addreſſed to an *Infidel Mathematician*, I do not find the leaſt attempt towards eſtabliſhing the evidence of our Chriſtian Faith.

I find therefore no other way of reconciling your Title-page with the ſubſtance of your diſcourſe, than by ſuppoſing you pretend to prove, that the object, principles, and inferences of the modern Analyſis are not more diſtinctly conceived, nor more evidently deduced, than religious Myſteries and point of Faith, *i. e.* that there is no more evidence and certainty in the modern Analyſis, than in the Chriſtian Religion. But how far you do honour to Chriſtianity, by entering into a compariſon between the evidence and certainty of the Religion taught us by our Saviour and his Apoſtles, and the evidence and certainty of the doctrine of Fluxions delivered by Sir *Iſaac Newton* and his followers, a doctrine in your opinion, *full of error, falſe reaſoning, manifeſt ſophiſms, fallacious and inconſiſtent ways of arguing, ſuch as would not be allowed of in Divinity,* I ſhall leave to your brethren the Divines to conſider.

Not to inſiſt therefore too long upon this, I ſhall now diſmiſs your Title-page, and ſhall come to your declared and avowed deſign, which, it muſt be owned, you have ſteadily purſued through your whole diſcourſe; to leſſen the reputation and authority of Sir *Iſaac Newton* and his followers, by ſhewing that they are not ſuch *great maſters of reaſon*, as they are generally preſumed to be; and to depreciate the ſcience they profeſs, by demonſtrating to the world that it is not of that clearneſs and certainty, as is commonly imagined.

You muſt excuſe us, Sir, if this deſign appears a very ſtrange one to us of the Univerſity, who plainly ſee of how great uſe Mathematical Learning is to mankind, not only to thoſe who make it a part of their ſtudies, but to all the reſt of the world, who without knowing any thing of Mathematicks, do yet daily and hourly reap the benefit of the inventions of Mathematical men in all parts of life, eſpecially in Mechanical arts, in Architecture, Civil, Naval, and Military, and in Navigation, upon which the proſperity and ſecurity of this Nation ſo much depends. Though we ſee and know, that the ſtudy of Mathematicks flouriſhes among us, as much as in any part of the world; and that our youth have as good aſſiſtance and opportunities for cultivating that ſcience, as are any where to be met with; yet we are far from thinking that too many of our ſtudents engage in this purſuit, or ſpend too much of their time in it. Believe me, Sir, the generality of youth have more need of the ſpur than the bridle, when they are to enter upon a ſtudy, that requires ſo laborious an attention, and ſo conſtant an application. Why then, would we ask, is the ſtudy of Mathematicks to be diſcouraged and undervalued? What could be the reaſon or motive for your engaging in ſuch a deſign?

I perceive your anſwer is ready. Mathematicians are Infidels, and make uſe of their reputation and authority, as being eſteemed *the greateſt Maſters of reaſon*, to pervert other perſons to infidelity. Really, Sir, this aſſertion of yours is new to us, and extremely ſurprizes us. We ſee nothing of it here, and cannot eaſily believe it is ſo any where elſe.

But admitting it for the preſent to be true; have you well conſidered whether it be for the advantage of Chriſtianity, to publiſh to the world, that a numerous ſet of learned men, and of ſuch learned men as are * *ſuppoſed to apprehend more diſtinctly, conſider more cloſely, infer more juſtly, conclude more accurately than other men*, do not believe the truth of the Chriſtian Religion? This, methinks, is an aſſertion fitter for an *Alciphron*, or a *Lyſicles* to make, than for a Chriſtian Divine. *Tell it not in Gath, publiſh it not in the ſtreets of Askalon.* Though the evidence and certainty of our holy Religion is ſo firmly eſtabliſhed, as not to be ſhaken by the arguments, much leſs by the reputation and authority of any unbelievers whatſoever; yet, I am afraid, it would be a great ſtumbling block to men of weak heads, if they were made to believe, that the juſteſt and cloſeſt reaſoners were generally Infidels. You know, Sir, it was a ſhrewd objection, tho' by no means a valid or concluſive one, that was made againſt our Saviour. *Have any of the High Prieſts, or of the Phariſees* (the men of greateſt learning, and eſteemed the *greateſt maſters of rea-*

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* *Analyſt*, p. 4.

ſon in thoſe days) *believed in him?* And if there were any room to renew this objection in our days, as conſidering the piety and zeal of the Chriſtian Prieſts of all degrees, and their conſtant and ſincere attachment to the intereſt of the Chriſtian Religion and to nothing elſe, ſurely there is not, it would indeed be a very perplexing one.

If we ſhould now make you a farther conceſſion, and ſuppoſe it not only to be true, but to be publickly and notoriouſly known, that the body of our Mathematicians are Infidels and enemies to Chriſtianity, what think you is the method to be taken? What other, you will ſay, but to ruin the reputation of our adverſaries? If we cannot attack them in their lives and converſations, we muſt at leaſt endeavour to ſhew that they are deſtitute of learning and reaſon. This will leſſen their authority among the people, will prevent their **miſleading unwary perſons in matters of the higheſt concernment*, and will take off † *that biaſs and deference for their judgment*, which cauſes weak ‡ *minds to ſubmit to their deciſions, where they have no right to decide.* Why really, Sir, this may do very well, if you are ſure of ſucceeding in your attempt. At leaſt it muſt be acknowledged, that you do *ſtare ſuper vias antiquas.* This is the very method which the *Odium Theologicum*, the intemperate zeal of Divines has always purſued, and has practiſed with great ſucceſs for many ages. But I muſt beg leave to obſerve, that it is a very different courſe from what was taken by *Jeſus Chriſt* and his Apoſtles on the like occaſion. *I thank thee, O Father, Lord of heaven and earth*, ſays our Saviour, *that thou haſt hid theſe things from the wiſe and prudent, and haſt revealed them unto babes.* And after his example, the great Apoſtle of the Gentiles, though he was fully ſenſible of the oppoſition he met with in preaching the Goſpel, from the *Greek* Philoſophers; yet did not think it neceſſary to enter the liſts with the Diſciples of *Plato* and the Stoicks, and to ſhew that they did not underſtand Geometry or Philoſophy; in order to leſſen their reputation, and thereby to take off from the weight of their oppoſition to Chriſtianity. He contents himſelf with obſerving, that *not many wiſe, not many learned are called*; though *the Greeks ſeek after wiſdom*, though *Chriſt crucified is unto the Greeks fooliſhneſs*, yet *the wiſdom of this world is fooliſhneſs with God.* Many more paſſages may be found in his Epiſtles to the ſame purpoſe.

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* *Analyſt*, p. 3. † p. 4. ‡ Ibid.

You ſee, Sir, that neither our Saviour, nor his Apoſtles, who were guided by his Holy Spirit, denied either wiſdom, or prudence, or learning to ſuch unbelievers as were really poſſeſſed of thoſe qualities: they well knew there was nothing to fear, that *there was no inchantment againſt Jacob, nor divination againſt Iſrael*, and that no human abilities or accompliſhments whatſoever could be able to withſtand the Divine authority, and the irreſiſtible evidence of the Religion they taught and defended.

It will be ſaid perhaps, this is too great a pattern for you to follow; though there was no fear that the abilities, the reputation and authority of any adverſaries whatſoever could withſtand our Saviour or his Apoſtles; yet what was no match for them, may poſſibly be too hard for you, unleſs you take proper means to prevent it. How, Sir! Has not our Saviour promiſed *to be with you unto the end of the world?* Has he not aſſured us, that even *the gates of hell ſhall not prevail againſt his Church?* After this declaration, is there any thing to fear? Can any means be neceſſary, but ſuch as he has recommended by his practice and example? Can calumny, and detraction, and artifice to leſſen the reputation of your adverſaries, be means fit to be uſed by the followers of St. *Paul*, and *of Jeſus Chriſt?* If any human means are requiſite, ſurely they ſhould be ſuch as are innocent and juſt, rather than this criminal method of leſſening or detracting from your opponents.

What are theſe? you will ſay. Why plainly, Sir, to fight your enemies with their own weapons, to endeavour to excel in thoſe arts with which they oppoſe you. St. *Paul made himſelf all things to all men, if thereby he might gain ſome.* If Mathematicians are ſuch dangerous adverſaries to Chriſtianity, let the Church of *England* take care of the education of her Clergy, let her write over her pulpits, as *Plato* is ſaid to have done over the entrance of his ſchool, οὐδεὶς ἀγεομέτρητος εἰσίτω, Let no man enter into orders, unleſs he be an able Mathematician. When this is done, Sir, let us ſee what Mathematical Infidel will dare to beard a Chriſtian Prieſt, as *great a maſter of reaſon* as himſelf, and armed beſides with his

Theological accompliſhments, and the authority of his ſacred Function, but above all with that ineffable and incomprehenſible Gift, which every one of you receive at your ordination.

Surely no man will think this too laborious a task for the zeal and piety of the Chriſtian Clergy to undertake, if the ſalvation of ſouls depends upon it. Do not we ſee how wholly they devote themſelves to this only end and purpoſe; how, neglecting and deſpiſing all the gratifications of ſenſe, all the allurements of worldly intereſt, of honour, riches and power, they allot every portion of their time that can poſſibly be ſpared from the neceſſary exerciſes of their function, to the better enabling themſelves to take care of the ſouls committed to their charge? Do not they for this purpoſe, with incredible pains and indefatigable induſtry, ranſack and make themſelves maſters of all the treaſures of antiquity ſacred and profane? Hence thoſe nervous and cogent arguments, that invincible power of reaſon, that reſiſtleſs force of eloquence which thunders from every pulpit, to withſtand the gainſayer, to reclaim the deluded, to confirm the wavering, to ſilence, confound and aſtoniſh the obſtinate, incorrigible Heretick and Infidel. And ſhall we, can we ſuppoſe, that a ſmall part of their time and pains will not be allowed for arming themſelves with proper weapons to encounter our Mathematical unbelievers? You, Sir, have already ſhewn them how eaſily this is to be done; ſince notwithſtanding your many and great avocations, both at home and in very diſtant regions, you have not only made your ſelf maſter of the moſt profound, the moſt obſcure and moſt difficult parts of that ſtudy; but by looking cloſer and more attentively into them than any body has ever done before you, have diſcovered their moſt ſecret defects, and have unravelled their moſt hidden and intricate fallacies. And I make no doubt but your great example will excite ſuch a ſpirit of piety and zeal, and induſtry among your brethren, that in time to come even *Euclid*, and *Archimedes*, and *Apollonius*, if they are judged to ſtand in the way and obſtruct the progreſs of Chriſtianity, will be ſhewn to abound as much with falſe and fallacious reaſoning, as Sir *Iſaac Newton.*

This, Sir, would be the innocent, the juſt and honourable method of encountring the adverſaries of our Religion; rather to meet them in the field, than to ſeek to nail their cannon, to blow up their ammunition, and ſet fire to their magazines: but if you chuſe the other, as ſafer, though leſs commendable, as eaſier to your brethren, though more laborious to your ſelf, I am content; and provided you go through with the undertaking, ſhall be ready to join with the Reverend Defenders of our faith, in a ſolemn chorus to your eternal honour and renown.

Unus homo nobis carpendo reſtituit rem:
Non ponebat enim rumores ante ſalutem:
Ergo magiſq; magiſq; viri nunc gloria claret.

Now therefore, Sir, we come to our main and principal point, to examine by what means, in order to prevent the danger that Chriſtianity is threatened with from our Mathematical Infidels, you propoſe to leſſen their reputation, and to ſhew that they are not ſuch juſt and cloſe reaſoners as is commonly imagined. I ſhall prove, ſay you, that the method of Fluxions invented by Sir *Iſaac Newton*, and implicitly received by all our Mathematicians, is built upon falſe and precarious principles.

This will do ſomething, I muſt confeſs, in as much as it will ſhew, that Mathematicians are men and liable to error. But if they are miſtaken in thinking the method of Fluxions to be a ſound and juſt one, will it follow from this, that in all the other parts of their ſcience they do not reaſon juſtly and accurately ? You know very well, Sir, that Fluxions are of very late invention, and that though they are a noble and uſeful part of Mathematicks, yet the whole of Mathematicks does not conſiſt in Fluxions. Mathematicians were always reckoned to be *great maſters of reaſon*, before Fluxions were invented. And as the uſe of Fluxions does by no means ſuperſede the doctrine of Geometry delivered by *Euclid*, *Apollonius*, and ſo many others of the great maſters both antient and modern, not to mention Algebra and all the other parts of the Mathematicks, againſt which you make no objection ; will it not follow that Mathematicians are ſtill to be eſteemed juſt and accurate reaſoners, though they have been led into ſome miſtakes in the method of Fluxions, by their implicit deference to the judgment of the Great Author of that invention ?

But farther, if it ſhould appear that there is no miſtake in that method, that the object of it is clearly conceived, that its principles are evident and certain, that the inferences drawn from them are juſtly deduced, that the whole doctrine is eſtabliſhed upon a clear, juſt, and ſolid foundation, and that it is you only who have been guilty of a moſt inexcuſable overſight, in which the meaneſt Mathematician you could have conſulted, would have immediately ſet you right: what then will become of your noble and well concerted project to leſſen the reputation of Mathematicians for the ſervice of Chriſtianity?

Believe me, Sir, our holy Religion ſtands in need of no ſuch attempts to ſerve it. *Non tali auxilio, nec defenſoribus iſtis.* The better reaſoners either Mathematicians, or any other ſort of men ſhall be, the more likely will they be to embrace truth wherever it ſhall be found, and conſequently the more likely to receive the truth and undoubted evidence of the Chriſtian Religion.

But pray, Sir, was it really and indeed the intereſt of Chriſtianity, and nothing elſe, that prompted you to this undertaking? Was that your only motive? The reaſon of my doubt is, that Sir *Iſaac Newton*, who by your own acknowledgment was not an Infidel, is no more ſpared in your performance, nay on the contrary is more ſeverely handled than any of his followers, whom you ſuppoſe to be Infidels. For beſides the charge brought againſt him and them in common, of the *want of good Geometry and good Logic*, *of error and falſe reaſoning*, you frequently repreſent him as *conſcious of the defect of his principles*, and yet endeavouring to impoſe upon and to miſlead his followers *by ſhifts, artifice and ſubtilty, by ambiguous terms, by puzzling, palliating and fallacious ways of proceeding*, &c. that is, you repreſent them only as weak men, deceived and impoſed upon; but him both as a weak and an ill man; as not only deceived, but a deceiver and an impoſtor. Did zeal for Chriſtianity move you to this? To treat in this manner a perſon whom you acknowledge to have been a believer of the Chriſtian Religion? I am afraid we muſt look out for ſome other motive, and in order to diſcover it, I ſhall take the liberty of conſidering a little what has been your conduct with regard to Mathematicians, not only ſince you have been informed of their being Infidels, but five and twenty years ago, when it may be reaſonably ſuppoſed you had heard and ſeen leſs of them, than you have lately done.

About that time you publiſhed an *Eſſay towards a new Theory of Viſion,* wherein you were pleaſed to inſert a great many ſevere cenſures upon Mathematicians, relating to their ignorance of the fundamental principles of their own ſcience. Had you then heard of their being Infidels? If ſo, why were many of thoſe cenſures greatly mollified, upon farther conſideration, in a new edition of that piece publiſhed the laſt year? And how comes a new charge of ignorance to be brought againſt them, on another account, at this time? Really, Sir, I can no way aſcribe all this to a zeal for Chriſtianity: but, if I may be allowed to make one hypotheſis, it will then be eaſy to account clearly and diſtinctly for every particular of your conduct. It is briefly this, that you have too great an opinion of your ſelf, and too mean a one of all other men. Hence, not content with the reputation you have deſervedly acquired of being a clear and juſt reaſoner, you can never reſt, unleſs you convince the world that all thoſe, who have hitherto been eſteemed the *greateſt maſters* *of reaſon*, are in this reſpect greatly inferior to Dr. *B—y*. Elſe why theſe attacks upon Mathematicians in general? Why are Dr. *Barrow* and Sir *Iſaac Newton* ſingled out from among the Mathematicians; and why Mr. *Locke* and Lord *Shaftsbury* among thoſe who are eſteemed clear and juſt reaſoners, though not Mathematicians, but to ſhew that Dr. *B——y* is a cloſer, a juſter, and more accurate reaſoner than any of theſe four Gentlemen not to be matched perhaps by four others, not only in *Great Britain*, but in the world? Were theſe Gentlemen all unbelievers? Yourſelf will not call them ſo; and if we except one, you will not in the leaſt ſuſpect them to be ſo. Why then are they all attacked in your writings? I cannot poſſibly ſurmiſe any other cauſe, but that they are all eſteemed *great maſters of reaſon*, and that you are diſpoſed to leſſen their reputation in order to exalt your own. Two of them you have been pleaſed to charge with contradiction, groſs, apparent, glaring contradiction, ſuch as were utterly unpardonable in a man who makes the leaſt pretence to reaſoning; ſuch as no body could poſſibly ſuſpect in thoſe great men, who had not a much greater opinion of his own abilities than of theirs; ſuch as no man who had the leaſt diſtruſt or diffidence of himſelf, could ever imagine them to be guilty of. I would ſay, Sir, that had you ſuppoſed it poſſible for your ſelf ever to fall into any miſtake; you would certainly, when you imagined Sir *Iſaac Newton* and Mr. *Locke* to have ſo manifeſtly contradicted themſelves, have looked over thoſe paſſages of their writings a ſecond time; you would have done this coolly, attentively and conſiderately; and had you done ſo, you would have found your ſelf in a groſs, evident miſtake; and would have ſeen them to be clearly and perfectly conſiſtent with themſelves. I muſt take leave to add, that your not doing ſo, does evidently prove the truth of the hypotheſis I laid down but now, that you have too good an opinion of your ſelf, and too mean a one of thoſe great men. One of theſe I ſhall of courſe be led to vindicate in the purſuit of my deſign; and as your miſtake about the other has a near relation to Geometry, before I cloſe this letter, I ſhall take up a little of your time in rectifying that likewiſe.

Having finiſhed my conſiderations upon the deſign, the uſefulneſs, prudence and juſtice

ſtice of your performance, and your motive for entering upon it; I come now to examine into the charge you have brought againſt our Mathematicians.

1. Of infidelity with reſpect to the Chriſtian Religion.
2. Of endeavouring to make others Infidels, and ſucceeding in thoſe endeavours, by means of *that deference* which is paid *to their judgment, as being preſumed to be of all men the greateſt maſters of reaſon.*
3. Of error and falſe reaſoning in their own ſcience.

Here, Sir, I muſt beg leave to ask, what proof you can bring of the two firſt articles of this accuſation? I have carefully peruſed your whole diſcourſe, and do not find the leaſt foundation for them, except only that you ſay, in your firſt page, *you are not a ſtranger to the authority that one* nameleſs Infidel Mathematician *aſſumes in things foreign to his profeſſion, nor to the abuſe that he, and too many more of the like character, are known to make of ſuch undue authority, to the miſleading of unwary perſons in matters of the higheſt concernment, and whereof his Mathematical knowledge can by no means qualify him to be a* *competent judge*; and that in your ſecond page, you tell us, *That this is one way of making Infidels, you are credibly informed.*

How ſay you, Sir, *you are not a ſtranger to it, it is known, you are credibly informed? Bene habet, nil plus interrogo.* Let us burn or hang up all the Mathematicians in *Great Britain*, or halloo the mob upon them to tear them to pieces every mother's ſon of them, *Tros Rutulusve fuat*, Laymen or Clergymen, tho' ſome of theſe, by their preaching, their writings, their lives and converſations, are thought to do honour to their order. Let us dig up the bodies of Dr. *Barrow* and Sir *Iſaac Newton*, and burn them under the gallows, and demoliſh the monuments erected to their memories. What, tho' the firſt be eſteemed one of the greateſt luminaries of the Chriſtian Church, and the other be acknowledged to have been a true believer, and to have given ſome of the ſtrongeſt and cleareſt proofs, that have ever been produced, of the goodneſs, wiſdom, and power of the Supreme Being? No matter: their followers are Freethinkers, Minute Philoſophers and Infidels, and labour to make others ſo. Dr. *B——y is not a ſtranger to it, it is known, he is credibly informed of it.*

For God's ſake, Sir, are we in *England* or in *Spain?* Is this the language of a Familiar, who is whiſpering an Inquiſitor againſt a ſingle Heretick, or of a Proteſtant Divine againſt a great number of Gentlemen profeſſing the Proteſtant Religion, and in a Proteſtant Country? There indeed ſuch an accuſation might be deſtructive to the perſons it fell upon, how innocent ſoever: but here, thanks be to God, and to our Religious and Civil Liberties, as no proof is brought againſt us, we have no more to do, but to aver our innocence, abſolutely to deny the charge, and plainly to tell you, that your *Informers*, how *credible* ſoever you may think them, are no other than a pack of baſe, profligate, and impudent Lyers.

This language may ſeem too warm: but ſo deteſtable and groundleſs a defamation will juſtify the uſe of it, And I muſt tell your ſelf, Sir, that tho' I can by no means think you to have been the author and inventor of ſo horrid a calumny; yet you can never be acquitted of extreme credulity in giving ear and belief to it; nor of an unpardonable injuſtice in publiſhing and propagating it, as far as in you lay, through the nation. Surely, if you had not a very ſtrong inclination to think and ſpeak ill of Mathematicians, you could not have done either the one or the other. For is it at all likely or probable, that a number of perſons, great part of whoſe employment has been to ſtudy Geometry, *an excellent Logic*, as you obſerve,* *where the definitions are clear, where the Poſtulata cannot be refuſed, nor the Axioms denied; where from the diſtinct contemplation and compariſon of figures, their properties are derived, by a perpetual well-connected chain of conſequences, the objects being ſtill kept in view, and the attention ever fixed upon them; whereby there is acquired an habit of reaſoning, cloſe and exact and methodical: which habit ſtrengthens and ſharpens the mind, and being transferred to other objects, is of general uſe in the inquiring after truth*; is it probable, or even poſſible, I would ſay, that a great number of perſons, who have *acquired this habit of reaſoning*, ſhould generally not ſee and comprehend the clear, the certain and undeniable evidence of the Chriſtian Religion? *Credat Judæus Apella.*

Nor can I find a grain more of probability in the ſecond article of your accuſation, than in the firſt. Suppoſe a Mathematician to be an Infidel, and to endeavour to make converts to Infidelity, will a deference be paid to his judgment in Divinity any more than in Law, or Phyſick, or any other ſcience foreign to his own profeſſion? Will his deciſions againſt the Chriſtian Religion paſs even upon weak and vulgar minds, becauſe he is thought to reaſon well in Geometry? *Id populus curat ſcilicet!* Dr. *Barrow* was a great and famous Mathematician: till Sir *Iſaac Newton* ſhone out,

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* *Analyſt*, p. 5.

Qui genus humanum ingenio ſuperavit, & omnes
Præſtrinxit, ſtellas exortus uti ætherius ſol,

Dr. *Barrow* was eſteemed the greateſt Geometrician in *England*: he was likewiſe a learned, ſound and Orthodox Divine; and yet the Arrians, the Socinians, the Quakers, and the ſeveral other ſects of Diſſenters are ſtill in being, though that *great Maſter of reaſon* both in Mathematicks and Divinity was of a contrary opinion to them all. Sir *Iſaac Newton* was a greater Mathematician than any of his contemporaries in *France*; no *Frenchman* will deny it: and yet I have not heard that the French Mathematicians are converted to the Proteſtant Religion by his authority, though Sir *Iſaac* was known to be a zealous Proteſtant. When this great man had any illneſs, he did not truſt to his own judgment for the cure of it, though ſo *great a maſter of reaſon*, but ſent for Dr. *Mead*, and ſubmitted himſelf intirely to his directions; and I ſuppoſe, upon occaſion, would have paid the ſame deference to his Lawyer. But I am aſhamed to inſiſt ſo long upon a thing ſo evident, and ſhall therefore now come to that article of your charge, which you ſeem to have much more at heart than the intereſt of the Chriſtian Religion, namely that of the errors and falſe reaſoning of Mathematicians in their own ſcience, or rather in that one part of their ſcience, which is commonly called the method of Fluxions.

Your objections againſt this method may, I think, all of them be reduced under theſe three heads.

1. Obſcurity of this doctrine.

2. Falſe reaſoning uſed in it by Sir *Iſaac Newton*, and implicitely received by his followers.

3. Artifices and fallacies uſed by Sir *Iſaac Newton*, to make this falſe reaſoning paſs upon his followers. I ſhall treat of theſe in their order.

1. Obſcurity of the doctrine of Fluxions.

It muſt be owned that this doctrine, as it is one of the moſt profound parts of Geometry, and perhaps the greateſt inſtance that has ever been given of the extent of human abilities, is not without its difficulties, and will doubtleſs ſeem obſcure to every unqualified or inattentive reader of Sir *Iſaac Newton*. But if a perſon duly furniſhed with the neceſſary previous knowledge of Geometry, ſhall peruſe his writings with that care and attention which the dignity and importance of the ſubject deſerves ; I do aſſure you, Sir, from my own experience, and that of many others whom I could name, that the doctrine may be clearly conceived and diſtinctly comprehended. If *your imagination is ſtrained and puzzled with it*, if it appears to you to contain *obſcure and inconceivable myſteries*, in ſhort, if you do not underſtand it, I tell you others do ; and you may do ſo too, if you will read it with due attention, and with a deſire of comprehending it, rather than an inclination to cenſure it.

You will ſay, perhaps, you have already done this, and find *the clear conception of it to be impoſſible, and appeal to the trial of eve-*

ry thinking reader. Pray, Sir, who are theſe *thinking readers* you appeal to? Are they Geometricians, or perſons wholly ignorant of Geometry? If the former, I leave it to them: if the latter, I aſk, how well are they qualified to judge of the method of Fluxions? I, you will ſay, have repreſented that method to them, and from what I have laid before them, they may eaſily judge of it. But have you fairly, and truly, and fully repreſented it? Have not you very much abridged what Sir *Iſaac Newton*, for the more eaſy and clear comprehenſion of his doctrine, has delivered more at large in thoſe * parts of his works from which your account of Fluxions is taken? Have you given any account of another ‡ part of his writings, where the foundation of this method is geometrically demonſtrated, and largely explained, and difficulties and objections againſt it are clearly ſolved? Have you not altered his expreſſions in ſuch a manner, as to miſlead and confound your readers, inſtead of informing them? Where do you find Sir *Iſaac Newton* uſing ſuch expreſſions as *the velocities of the velocities, the*

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* *Introd. ad Quadr. Curv. & Princ. Phil. Math. lib.* 2. *lemm.* 2.

‡ *Princ. Phil. Math. lib.* 1. *Sect.* 1.

ſecond, third and fourth velocities, the incipient celerity of an incipient celerity, the naſcent augment of a naſcent augment? Is this the true and genuine meaning of the words, *fluxionum mutationes magis aut minus celeres?* Believe me, Sir, it is very eaſy by ſuch pious arts as theſe, to make any doctrine appear abſurd even to a *thinking reader*; unleſs inſtead of truſting to the repreſentation of an adverſary, he will take the pains to conſult the Author that is cenſured. This therefore I ſhall adviſe both your *thinking reader* and your ſelf to do, and ſhall betake myſelf to the conſideration of the next head of your objections.

2. Falſe reaſoning uſed in the method of Fluxions by Sir *Iſaac Newton*, and implicitely received by his followers.

Of this you produce two inſtances, both relating to a point which you juſtly call fundamental, namely the rule for finding the Fluxion of the rectangle of two flowing quantities; one of which inſtances is taken from the *Principia Philoſ.* and the other from the book of Quadratures.

† The objection you make to the firſt of theſe is, that the Author, in order to

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† *Analyſt. p.* 16, 17.

find the moment or increment of the rectangle AB, does not take the ſides $A + a$, and $B + b$, as increaſed by their whole moments, a and b, which you ſay is *the direct and true method*; but inſtead thereof uſes *the illegitimate and indirect method* of ſuppoſing the ſides A and B to be *deficient or leſſer by one half of their moments*, as $A - \frac{1}{2} a$, and $B - \frac{1}{2} b$, and then finding what the increment of the rectangle will be, when thoſe ſides come to be *increaſed by the other two halves of their moments*, as $A + \frac{1}{2} a$, and $B + \frac{1}{2} b$.

By the former method the increment of the rectangle AB would come out $aB + bA + ab$: by the latter it is $aB + bA$; and the difference between theſe two is the rectangle ab. Here you are pleaſed exceedingly to inſult and triumph over the great Author of this method and all his followers, as being very much at a loſs how to get rid of this ſame rectangle ab. Why, Sir, ſuppoſe they cannot in geometrical rigour get rid of it, but that it muſt continue to be a part of the increment of the rectangle AB; and yet Mathematicians reject it, and inſtead of the rigorous geometrical increment $aB + bA + ab$, they uſe only $aB + bA$:

pray, what is it you would infer from this? Will it follow that they do not *proceed ſcientifically*, that they are *at a loſs how to conduct themſelves*, that they *don't ſee their way diſtinctly*, that they *proceed blindfold?* Will you ſay that they do not exactly ſee the conſequence of this omiſſion, through their whole proceeding, and do not certainly and clearly know how far their concluſion will be affected by it? Do not they know that in eſtimating any finite quantity how great ſoever, propoſed to be found by the method of Fluxions, a globe, ſuppoſe, as big as that of the earth, or, if you pleaſe, of the ſun, or of the whole planetary ſyſtem, or even the orb of the fixed ſtars; do not they know, I ſay, and are they not able clearly and invincibly to demonſtrate that, in ſo immenſe a magnitude, this omiſſion ſhall not cauſe them to deviate from the truth ſo much as a ſingle pin's head, nay not the thouſandth, not the millionth part of a pin's head? How then can it be ſaid that *they proceed without clearneſs and without ſcience*, and don't know what they are doing, nor whither this method is carrying them?

I ſhall here beg leave, for the ſake of

readers leſs mathematically qualified, to put a very eaſy and familiar caſe. Suppoſe two Arithmeticians to be diſputing whether vulgar fractions are to be preferred to decimal; would it be fair in him who is for expreſſing the third part of a farthing by the vulgar fraction $\frac{1}{3}$, to affirm that his antagoniſt proceeded blindfold, and without knowing what he did, when he pretended to expreſs it by 0. 33333 &c. becauſe this expreſſion did not give the rigorous, exact value of one third of a farthing? Might not the other reply that, if this expreſſion was not rigorouſly exact, yet it could not be ſaid he *proceeded blindfold, or without clearneſs and without ſcience* in uſing it, becauſe by adding more figures he could approach as near as he pleaſed, and wherever he thought fit to ſtop, he could clearly and diſtinctly find and demonſtrate how much he fell ſhort of the rigorous and exact value? Might not he further ſay that as the &c. implied all the poſſible repetitions of the figure 3, even to infinity, therefore his expreſſion did not differ by any the leaſt aſſignable quantity from the other value, $\frac{1}{3}$; and that as he knew and clearly conceived that it did ſo, he could not juſtly be ſaid to be in any error, much leſs to act in the dark, when he uſed that expreſſion?

Having obſerved that this omiſſion, or error as you are pleaſed to call it, in rejecting the rectangle $a\,b$, is at moſt ſuch an one as can cauſe no aſſignable difference, how ſmall ſoever, in the concluſions drawn from the method of Fluxions; and that Mathematicians in committing this error, do nevertheleſs proceed ſcientifically and with their eyes open, as having a clear and diſtinct view not only of the original error, but of the effect of it both in every ſtep they take, and in all the concluſions they draw from this method; I ſhall now for your farther ſatisfaction proceed to examine, whether in reality it be any error at all; and whether $a\,B + b\,A$ be not in ſtrict, geometrical rigour, the true fluxion, moment, or increment of the rectangle $A\,B$. And as you ſay *the foreign Mathematicians are ſuppoſed by ſome, even of our own, to proceed in a manner, leſs accurate perhaps and geometrical, yet more intelligible*, than that of Sir *Iſaac Newton*; I ſhall firſt conſider what courſe they take in rejecting the rectangle $a\,b$, and ſhall then proceed to Sir *Iſaac Newton*'s way of excluding it.

The famous *Marquis de l'Hoſpital,* whom I the rather follow, becauſe he is thought to have written upon this ſubject with greater perſpicuity than any other foreign Mathematician, as alſo becauſe you expreſsly quote him, after he has found the fluxion of the rectangle * $x\, y$, compoſed of the two variable quantities x and y, to be $y\, d\, x + x\, d\, y + d\, x\, d\, y$; does afterwards reject the rectangle $d\, x\, d\, y$, and thereby leave only $y\, d\, x + x\, d\, y$ for the fluxion of $x\, y$. Upon which you *obſerve,* † *as to the method of getting rid of this quantity* $d\, x\, d\, y$, equivalent to the rectangle $a\, b$ of Sir *Iſaac Newton, that it is done without the leaſt ceremony.* Is this true? Does not the Marquis, in this very propoſition ‡ quoted by you, and at the very inſtant that he rejects that quantity, give this reaſon for it, that it is infinitely ſmall with reſpect to the other terms $x\, d\, y$ and $y\, d\, x$? Does he not immediately after give a demonſtration of its being ſo? And does he not, to ſhew that he has a right to

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* *Analyſe* des *infiniment petits. Prop.* 2.

† *Analyſt. p.* 28. ‡ *ibid. p.* 26.

reject it upon that reaſon, plainly and expreſsly refer his reader to his firſt *poſtulatum* or ſuppoſition? Does he not in that *poſtulatum* expreſsly require it ſhould be allowed him, *that a quantity, which is augmented or diminiſhed by another quantity infinitely leſs than the firſt, may be conſidered as if it continued the ſame*, i. e. had received no ſuch augmentation or diminution? Is not this plainly the caſe of the quantity $x\,d\,y + y\,d\,x$, augmented or diminiſhed by the quantity $d\,x\,d\,y$? Have you then done fairly and juſtly by this Great Man, in concealing all this from your *thinking readers* to whoſe judgment you refer your ſelf, and in telling them that *this quantity is rejected without the leaſt ceremony?*

You will tell me perhaps, that you do not allow of this *poſtulatum.* Why then you muſt not read the *Marquis de l'Hoſpital*'s book. The *poſtulatum* is placed at the very beginning of it, as an expreſs declaration to his readers, that unleſs that be allowed him, he will not undertake to demonſtrate what follows. If you admit his *poſtulatum*, you will find him proceed clearly and evidently and like a Mathematician in rejecting the quantity $d\,x\,d\,y$: if not, you have no right to attack his propoſition founded upon that *poſtulatum*, but only to give your reaſons againſt the *poſtulatum* itſelf. And thus much in vindication of my firſt maſter, that great and clear-headed Geometrician the *Marquis de l'Hoſpital*, whoſe only misfortune it was to have met with muddy waters, and not to have drunk of the fountain itſelf.

I come now to conſider what courſe Sir *Iſaac Newton* himſelf has taken to avoid this formidable rectangle *a b*, this fatal rock, this *Biſhop and his Clerks*, that threatens deſtruction to him and all his followers. And here, Sir, in order to give your reaſoning its full force, I ſhall tranſcribe the greateſt part of the ninth ſection of your diſcourſe, after which I ſhall do the ſame juſtice to Sir *Iſaac Newton*, by giving his demonſtration in his own words. Your ninth ſection begins thus.

" Having conſidered the object, I pro-
" ceed to conſider the principle of this
" new Analyſis by Momentums, Fluxions,
" or Infiniteſimals; wherein if it ſhall ap-
" pear that your capital points, upon which
" the reſt are ſuppoſed to depend, include
" error and falſe reaſoning; it will then
" follow

“ follow that you, who are at a loſs to conduct your ſelves, cannot with any decency ſet up for guides to other men. The main point in the method of Fluxions is to obtain the fluxion or momentum of the rectangle or product of two indeterminate quantities. Inaſmuch as from thence are derived rules for obtaining the Fluxions of all other products and powers; be the coefficients or the indexes what they will, integers or fractions, rational or ſurd. Now this fundamental point one would think ſhould be very clearly made out, conſidering how much is built upon it, and that its influence extends throughout the whole Analyſis. But let the reader judge. This is given for demonſtration. * Suppoſe the product or rectangle AB increaſed by continual motion: and that the momentaneous increments of the ſides A and B are a and b. When the ſides A and B were deficient, or leſſer by one half of their moments, the rectangle was $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, i. e. $AB - \frac{1}{2}aB - \frac{1}{2}bA + \frac{1}{4}ab$. And as ſoon as the ſides A and B are increaſed by the other two halves of

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* *Philoſ. Nat. Princ. Math. Lib. II. Lemm.* 2.

“ their moments, the rectangle becomes
“ $\overline{A + \frac{1}{2} a} \times \overline{B + \frac{1}{2} b}$, or $A B + \frac{1}{2} a B$
“ $+ \frac{1}{2} b A + \frac{1}{4} a b$. From the latter rectangle
“ ſubduct the former, and the remaining
“ difference will be $a B + b A$. Therefore
“ the increment of the rectangle generated
“ by the intire increments a and b
“ is $a B + b A$. Q. E. D. But it is plain
“ that the direct and true method to obtain
“ the moment or increment of the rectangle
“ $A B$, is to take the ſides as increaſed by
“ their whole increments, and ſo multiply
“ them together, $A + a$ by $B + b$, the
“ product whereof $A B + a B + b A +$
“ $a b$ is the augmented rectangle; whence
“ if we ſubduct $A B$, the remainder $a B$
“ $+ b A + a b$ will be the true increment
“ of the rectangle, exceeding that which
“ was obtained by the former illegitimate
“ and indirect method by the quantity
“ $a b$. And this holds univerſally be the
“ quantities a and b what they will, big or
“ little, finite or infiniteſimal, increments,
“ moments, or velocities. Nor will it avail
“ to ſay that $a b$ is a quantity exceeding
“ ſmall: Since we are told * that *in rebus*
“ *mathematicis errores quam minimi non ſunt*

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* *Introduct. ad Quadrat. Curv.*

“ *contemnendi.* Such reaſoning as this, for demonſtration, nothing but the obſcurity of the ſubject could have encouraged or induced the great author of the fluxionary method to put upon his followers, and nothing but an implicite deference to authority could move them to admit. The caſe indeed is difficult. There can be nothing done till you have got rid of the quantity $a\,b$. In order to this the notion of Fluxions is ſhifted: It is placed in various lights: Points which ſhould be as clear as firſt Principles are puzzled; and terms which ſhould be ſteadily uſed are ambiguous. But notwithſtanding all this addreſs and skill the point of getting rid of $a\,b$ cannot be obtained by legitimate reaſoning.

It is now time to hear Sir *Iſaac Newton.*

Princip. Lib. II. Lemm. 2. Caſ. 1. “ Rectangulum quodvis motu perpetuo auctum $A\,B$, ubi de lateribus A & B deerant momentorum dimidia $\frac{1}{2}\,a$ and $\frac{1}{2}\,b$, fuit $A - \frac{1}{2}\,a$ in $B - \frac{1}{2}\,b$, ſeu $A\,B - \frac{1}{2}\,a\,B - \frac{1}{2}\,b\,A + \frac{1}{4}\,a\,b$; & quamprimum latera A & B alteris momentorum dimidiis aucta ſunt, evadit $A + \frac{1}{2}\,a$ in $B + \frac{1}{2}\,b$, ſeu $A\,B + \frac{1}{2}$

“ $aB + \frac{1}{2}bA + \frac{1}{4}ab$. De hoc rectan-
“ gulo ſubducatur rectangulum prius, &
“ manebit exceſſus $aB + bA$. Igitur late-
“ rum incrementis totis a & b generatur rec-
“ tanguli incrementum $aB + bA$. Q. E. D.

Having now fairly laid before my reader what both your ſelf and Sir *Iſaac Newton* have delivered upon this ſubject, I come to examine which of you is in the right.

In the firſt place, I find you take it for granted that what Sir *Iſaac Newton* is here endeavouring to find, by ſuppoſing the ſides A and B firſt to want one half of their moments, and afterwards to have gained the other halves of their moments, is the increment of the rectangle AB. In this I conceive you are miſtaken. For neither in the demonſtration itſelf, nor in any thing preceding or following it, is any mention ſo much as once made of the increment of the rectangle AB. On the contrary it plainly appears that what he endeavours to obtain by theſe ſuppoſitions, is no other than the increment of the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, and you muſt own that he takes *the direct and true method to obtain it*. But you will ſay, is it not the buſineſs of this lemma to determine the moments of flowing quantities? And is it not the deſign of Caſe 1 to determine the moment of the rectangle AB? I anſwer that it is ſo: but that rigorouſly ſpeaking the moment of the rectangle AB, is not, as you ſuppoſe, the increment of the rectangle AB; but it is the increment of the rectangle $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$. In order to clear up this point, I muſt obſerve,

1. That the word moment is uſed by Sir *Iſaac Newton* and your ſelf to ſignifie indifferently either an increment, or a decrement.

2. That $aB + bA + ab$ is by you demonſtrated to be the true increment of the rectangle AB.

3. That $aB + bA - ab$ is the true decrement of the ſame rectangle AB; as plainly appears upon taking the ſame *true and direct method* for finding the decrement, as you have uſed for finding the increment.

Now, Sir, I would humbly beg leave to inquire of you, who ſee ſo much more clearly into theſe matters than Sir *Iſaac Newton* or any of his followers; which of theſe two Quantities, $aB + bA + ab$ and $aB + bA - ab$, you will be pleaſed to call the moment of the rectangle AB? *The* *caſe indeed is difficult,* The difference between them is no leſs than $2\,ab$, juſt the double of that ſame ab, which has given us all ſo much trouble; and yet each of them plead an equal right to the title of moment. So equal a one, that, though I am very ſenſible of your *addreſs and skill,* yet there ſeems to be no poſſibility of deciding the controverſy between them *by legitimate reaſoning.* I ſee but two ways of doing it. One is that they ſhould toſs up croſs or pile for the title: Or if that be thought too boyiſh and unbeſeeming the Gravity of Mathematical quantities, they muſt even end the diſpute in an amicable manner, and without claiming any preference one of another, agree that they make two moments between them. Then, Sir, I apprehend the caſe will ſtand thus: $aB + bA + ab + aB + bA - ab$ making twice the moment of the rectangle AB; it follows that $aB + bA$ will make the ſingle moment of the ſame rectangle.

You ſee, Sir, after all the pains you have taken, this affair comes out, even upon your own conceſſions, juſt as Sir *Iſaac Newton* and his followers would have it. Believe me, there is no remedy. You muſt acquieſce.

Only, if it may be any Satisfaction to you to know why Sir *Iſaac* took this indirect way of finding the increment of $\overline{A - \frac{1}{2} a} \times \overline{B - \frac{1}{2} b}$, inſtead of proceeding directly to find the moment of the rectangle $A\ B$, I ſhall be ready to oblige you as far as can be expected from one of thoſe, who *have ſhown themſelves more eager in applying his method, than accurate in examining his principles.*

The final cauſe or motive to this proceeding, I find, is not unknown to you; you ſay *it is very obvious,* meaning, I ſuppoſe, that thereby it was intended to exclude this ſame troubleſome rectangle *a b.* Why truly, Sir, in a book of ſtrict demonſtration, as Sir *Iſaac Newton* intended his *Principia* ſhould be, it was certainly more proper to exclude that quantity, ſo as not to ſuffer it to appear, than firſt to introduce it into the reader's view and then to reject it.

You add that *it is not ſo obvious or eaſy to explain a juſt and legitimate reaſon for it, or ſhew it to be Geometrical.* How far it may be obvious or eaſy to aſſign ſuch a reaſon, I will not diſpute: though I am apt to think that what is eaſy to me, cannot be difficult to other perſons, provided they uſe the ſame endeavours to find the truth, as I have done. Now, I apprehend the reaſon of this proceeding of Sir *Iſaac Newton* to be the following very plain one: That in order to find the moment of the rectangle *A B*, it is more conſonant to ſtrict Geometrical rigour to take the increment of the rectangle $\overline{A - \frac{1}{2} a} \times \overline{B - \frac{1}{2} b}$, than to take the increment of the rectangle *A B* itſelf. And if I can make this appear, you muſt allow that he had a *juſt and legitimate reaſon for proceeding* as he did.

You know very well that the moment of the rectangle *A B* is proportional to the velocity of that rectangle, with which it alters, either in increaſing, or in diminiſhing. Now, I aſk, in Geometrical rigour what is properly the velocity of this rectangle? Is it the velocity with which the rectangle from *A B* becomes $\overline{A + a} \times \overline{B + b}$; or the velocity with which from *A B* it becomes $\overline{A - a} \times \overline{B - b}$? I find my ſelf exactly in the caſe of the Aſs between the two bottles of hay: I ſee no reaſon, nor poſſibility of a reaſon to determine me either one way, or the other. But methinks I hear the venerable Ghoſt of Sir *Iſaac Newton* whiſper me, that the velocity I ſeek for, is neither the one nor the other of theſe, but is the velocity which the flowing rectangle has, not while it is greater or leſs than $A\,B$, neither before, nor after it becomes $A\,B$, but at that very inſtant of time that it is $A\,B$. In like manner the moment of this rectangle is neither the increment from $A\,B$ to $\overline{A+a}\times\overline{B+b}$; nor is it the decrement from $A\,B$ to $\overline{A-a}\times\overline{B-b}$: It is not a moment common to $A\,B$ and $\overline{A+a}\times\overline{B+b}$, which may be conſidered as the increment of the former, or as the decrement of the latter: Nor is it a moment common to $A\,B$ and $\overline{A-a}\times\overline{B-b}$, which may be conſidered as the decrement of the firſt, or as the increment of the laſt: But it is the moment of the very individual rectangle $A\,B$ itſelf, and peculiar to that only; and ſuch as being conſidered indifferently either as an increment or decrement, ſhall be exactly and perfectly the ſame. And the way to obtain ſuch a moment is not to look for one lying between $A\,B$ and $\overline{A+a}\times\overline{B+b}$; nor to look for one lying between $A\,B$ and $\overline{A-a}\times\overline{B-b}$:

that is, not to ſuppoſe *A B* as lying at either extremity of the moment; but as extended to the middle of it; as having acquired the one half of the moment, and as being about to acquire the other; or as having loſt one half of it, and being about to loſe the other. And this is the method Sir *Iſaac Newton* has taken in the demonſtration you except againſt.

What ſay you, Sir? Is this *a juſt and legitimate reaſon* for Sir *Iſaac*'s proceeding as he did? I think you muſt acknowledge it to be ſo. For even if you ſhould ſtill have any doubt whether his proceeding be rigorouſly Geometrical; yet you cannot but confeſs that whether moments be conſidered as infinitely ſmall, or as finite quantities, his method approaches nearer to Geometrick rigour, than that which you propoſe. I think likewiſe you cannot but be ſenſible of great want of caution in your own proceeding; inaſmuch as that quantity, which Sir *Iſaac Newton* through this whole Lemma, and all the ſeveral caſes of it, conſtantly calls a moment, without confining it to be either increment or decrement, is by you inconſiderately, and arbitrarily, and without any ſhadow of reaſon given, ſuppoſed and determined to be an increment. And this, Sir, naturally leads me to give you a piece of friendly advice, which you ſeem to ſtand much in need of. It is that, whenever you take it into your head to criticiſe upon Sir *Iſaac Newton*'s writings, you firſt examine and weigh every word he uſes; and if you tranſlate him, keep cloſely to his expreſſion. Believe me, this Great Man, among his other extraordinary indowments, had a peculiar ſagacity in foreſeeing objections, as well as an averſion to diſputing. To theſe two qualities accompanied with extreme humanity and condeſcenſion it is owing, that he uſes ſuch accuracy in his expreſſion, that an intelligent and attentive reader can never miſtake him; and that he does of himſelf firſt propoſe, and then remove ſuch difficulties, as may naturally ariſe in the minds of even candid and judicious perſons, who are not yet maſters of the ſubject he treats of. But as for the *Homines ſtolidi & ad depugnandum parati*, he contents himſelf with obſerving that prudent caution in every word he uſes, that as they ſhall find nothing to miſlead them, ſo on the other hand, if they undeſervedly and unadviſedly attack him, they ſhall certainly and unavoidably *induere ſe in ſtimulos latentes,* and expoſe themſelves to the ſcorn and contempt of every unprejudiced obſerver.

This great example, which in any the loweſt degree to imitate is the higheſt honour I can ever arrive at, or even deſire, moves me to propoſe and remove an objection which may poſſibly ariſe in your mind, and hinder you from acquieſcing in one part of what I have juſt now laid before you. It is that I have ſuppoſed the rectangle $A\,B$ extended to the middle of its moment; as having acquired the one half of it, and being about to acquire the other; or as having loſt one half of it, and being about to loſe the other. You may ſay this is ſtrictly and exactly true in reſpect of the ſides of that rectangle; which ſides, from $A - \frac{1}{2}a$ and $B - \frac{1}{2}b$, are become A and B; and are about to become $A + \frac{1}{2}a$ and $B + \frac{1}{2}b$: but that it is not equally true of the rectangle compoſed of thoſe ſides, which from $\overline{A - \frac{1}{2}a} \times \overline{B - \frac{1}{2}b}$, or $A\,B - \frac{1}{2}a\,B - \frac{1}{2}b\,A + \frac{1}{4}a\,b$, is become $A\,B$; and is about to become $\overline{A + \frac{1}{2}a} \times \overline{B + \frac{1}{2}b}$, or $A\,B + \frac{1}{2}a\,B + \frac{1}{2}b\,A + \frac{1}{4}a\,b$: ſince the part of the moment which $A\,B$ is ſuppoſed to have gained, namely $\frac{1}{2}\ aB + \frac{1}{2}\ bA - \frac{1}{4}\ ab$, is not equal to that part of the moment which is about to be gained, namely $\frac{1}{2}\ aB + \frac{1}{2}\ bA + \frac{1}{4}\ ab$; the difference between them being $\frac{1}{2}\ ab$. In anſwer to this I reply, that theſe two quantities, $\frac{1}{2}\ aB + \frac{1}{2}\ bA - \frac{1}{4}\ ab$, and $\frac{1}{2}\ aB + \frac{1}{2}\ bA + \frac{1}{4}\ ab$, ſo long as a and b are finite quantities, are undoubtedly unequal; but that the more a and b are diminiſhed, by ſo much nearer will theſe quantities approach to an equality; and if a and b are diminiſhed *ad infinitum*, the two quantities will then be perfectly equal. See this demonſtrated *Princip. Lib. I. Sect.* 1. *Lemm.* 1. Which *Lemma*, for your own ſake and mine, I could wiſh you had conſulted ſooner.

Laſtly, to remove all ſcruple and difficulty about this affair, I muſt obſerve, that the moment of the rectangle AB, determined by Sir *Iſaac Newton*, namely $aB + bA$, and the increment of the ſame rectangle, determined by yourſelf, namely $aB + bA + ab$, are perfectly and exactly equal, ſuppoſing a and b to be diminiſhed *ad infinitum*; and this by the *Lemma* juſt now quoted.

I now come to your ſecond inſtance of falſe reaſoning, which you take from the

Book of Quadratures; and paſſing by the *Lemma* you ſo gravely lay down to ſhew, that when two contrary ſuppoſitions are made, nothing can be inferred from either of them; as a truth that no School-boy can be ignorant of; I ſhall here tranſcribe this inſtance of falſe reaſoning as you give it, with your obſervations upon it.

* " Let the quantity x flow uniformly,
" and be it propoſed to find the Fluxion of
" $\overset{n}{x}$. In the ſame time that x by flowing
" becomes $x + o$, the Power x^{n} becomes
" $\overline{x+o}|^{n}$, *i. e.* by the method of infinite Se-
" ries $x^{n} + nox^{n-1} + \frac{nn-n}{2}oox^{n-2}$ + &c.
" and the increments o and $nox^{n-1} + \frac{nn-n}{2}$
" oox^{n-2}. + &c. are one to another as 1 to
" $nx^{n-1} + \frac{nn-n}{2}ox^{n-2}$ + &c. Let now
" the increments vaniſh, and their laſt pro-
" portion will be 1 to nx^{n-1}. But it ſhould
" ſeem that this reaſoning is not fair or con-
" cluſive. For when it is ſaid, let the incre-

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* *Analyſt p.* 20.

" ments vaniſh, i. e. let the increments be " nothing, or let there be no increments, the " former ſuppoſition that the increments " were ſomething, or that there were incre- " ments, is deſtroyed, and yet a conſequence " of that ſuppoſition, i. e. an expreſſion " got by virtue thereof, is retained. Which, " by the foregoing *Lemma*, is a falſe way " of arguing. Certainly when we ſuppoſe " the increments to vaniſh, we muſt ſup- " poſe their proportions, their expreſſions, " and every thing elſe derived from the " ſuppoſition of their exiſtence to vaniſh " with them.

You are pleaſed to go on for ſome number of pages, *to make this point plainer*, *to unfold the reaſoning*, and *to propoſe it in a fuller light*. But I think we may as well ſtop here. You have already ſo fully *unfolded* it, that if this be the way of reaſoning of our Mathematical Infidels, I pronounce our Religion out of all danger from that quarter. From this time our Reverend Clergy may ſleep in quiet, and be under as little apprehenſion from the unbelieving Analyſt, as from the moſt ignorant of the Popiſh Monks, the moſt ſtupid of the Jewiſh Rabbi's, or the moſt empty and contemptible praters among the Minute Philoſophers. I have only one doubt upon me. Pray, Sir, are you very ſure that this is the real doctrine of Sir *Iſaac Newton?* Are you abſolutely certain you have not miſtaken him? You ſeem, I muſt confeſs, to be exceedingly cautious, you blame others for not being *accurate in examining his Principles*, you talk of *preventing all poſſibility of miſtaking you*, and you treat him and his followers in ſuch a manner, that you are to expect no quarter from them in caſe of ill ſucceſs. And yet this is ſo great, ſo unaccountable, ſo horrid, ſo truly Bœotian a blunder, that I know not how to think a *Great Genius*, a *Newton* could be guilty of it. For God's ſake let us examine it once more. *Evaneſcant jam augmenta illa, let now the increments vaniſh, i. e. let the increments be nothing, or let there be no increments.* Hold, Sir, I doubt we are not right here. I remember Sir *Iſaac Newton* often uſes the terms of *momenta naſcentia* and *momenta evaneſcentia.* I think I have ſeen you likewiſe ſeveral times uſing the like terms of naſcent and evaneſcent increments. Alſo, if I am not miſtaken, both he and you conſider a naſcent, or evaneſcent moment, an increment or decrement,

crement, as the ſame quantity under different circumſtances; ſometimes as in the point of beginning to exiſt, and other times as in the point of ceaſing to exiſt. From this methinks it ſhould follow that the two expreſſions ſubjoined, will be perfectly equivalent to each other.

Naſcantur *jam augmenta illa, & eorum ratio* prima *erit*

Evaneſcant *jam augmenta illa & eorum ratio* ultima *erit*

The meaning of the firſt can poſſibly be no other than to conſider the firſt proportion between the naſcent augments, in the point of their beginning to exiſt. Muſt not therefore the meaning of the latter be to conſider the laſt proportion between the evanaſcent augments, in the point of evaneſcence, or of their ceaſing to exiſt? Ought it not to be thus tranſlated, Let the augments now become evaneſcent, let them be upon the point of evaneſcence? What then muſt we think of your interpretation, *Let the increments be nothing, let there be no increments?* Do not the words *ratio ultima* ſtare us in the face, and plainly tell us that though there is a laſt proportion of evaneſcent increments, yet there can be no proportion of increments which are nothing, of increments which do not exiſt? I believe, Sir, every thinking perſon will acquit Sir *Iſaac Newton* of the groſs overſight you aſcribe to him, and will acknowledge that it is your ſelf alone, who have been guilty of a moſt palpable, inexcuſable, and unpardonable blunder. I now come to the third head of your objections.

3. Arts and fallacies uſed by Sir *Iſaac Newton* to make his falſe reaſoning paſs upon his followers.

On this head I ſhall not need to take up much of your time, becauſe having already fully proved that Sir *Iſaac Newton* was not guilty of falſe reaſoning in the inſtances you alledge, I ſuppoſe no body will think he had any occaſion to make uſe of arts and fallacies to impoſe upon his followers. But you have one obſervation upon this head, which is ſo very ſingular, that I cannot but think it worthy of particular conſideration. *Conſidering*, * ſay you, *the various arts and devices uſed by the Great Author of the Fluxionary method: in how many lights he placeth his Fluxions: and in what different ways he*

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* *Analyſt* p. 27.

attempts to demonſtrate the ſame point: one would be inclined to think he was himſelf ſuſpicious of the juſtneſs of his own demonſtrations: and that he was not enough pleaſed with any one notion ſteadily to adhere to it. Thus much at leaſt is plain, that he owned himſelf ſatisfied concerning certain points, which nevertheleſs he could not undertake to demonſtrate to others. Really, Sir, this ſeems to be very hard uſage. Sir *Iſaac Newton* has made a new and great diſcovery, by which he has not only out-done all the Geometricians that ever went before him, but can enable ſuch ordinary proficients in Mathematicks, as you and I, to ſurpaſs all the great maſters of antiquity: He is ſo good as to inſtruct us in this method; and becauſe it requires ſome pains and diſcernment to comprehend it rightly, he ſets it in ſeveral various lights, that by means of ſome of theſe we may not fail of underſtanding it. Pray, Sir, have you and I any reaſon to complain of this? For my part, I think myſelf greatly obliged to him for his condeſcenſion: If he had not taken ſo much pains to explain his doctrine, I doubt I ſhould never have underſtood it. But, for God's ſake, what is it you are offended at, who do not ſtill underſtand him! You are all in the dark, and yet are angry at his giving you ſo much light. Surely the fault is not in Sir *Iſaac Newton*, but in your own eyes.

> So thick a drop ſerene has quench'd their
>   orbs,
> Or dim ſuffuſion veil'd.

But is not he *himſelf*, ſay you, *ſuſpicious of the juſtneſs of his own demonſtrations?*

Pray, Sir, when a Divine is inſtructing his hearers in a weighty and important point of Religion, if from a deſire that every one ſhould perfectly underſtand him, he is at the pains to uſe ſeveral arguments, and to ſet his Doctrine in various lights; would it be reaſonable, or juſt, or grateful in any of his auditors to infer from this, that the Preacher was ſuſpicious of the juſtneſs of his own reaſoning? When you, after all the demonſtrations that had been given of the being of a God, by the learned Fathers of the Church, and by the wiſeſt of the Philoſophers of all ages, thought fit to introduce that new and ſingular one of a *Viſual Language*, would it be fair in me to ſuppoſe that you were ſuſpicious of all the former proofs of the exiſtence of a Deity, and left that great and important truth to depend upon a metaphorical argument? Surely one argument may be juſt, and concluſive, and perfectly ſatisfactory to him that uſes it; and yet the matter treated of may be of that difficulty, or of that dignity and importance, as not only to admit of, but to require ſeveral others for the inſtruction and conviction of his hearers. And thus much may ſuffice for your third and laſt head of objections againſt Sir *Iſaac Newton* and his followers: Only before I conclude I muſt adviſe you to correct one word in your extract from his * Letter to Mr. *Collins*, *Nov.* 8, 1676. or rather to give up that extract intirely, as being of no manner of ſervice to you. There is a great deal of difference between ſaying I cannot undertake to prove a thing, and I will not undertake it. Sir *Iſaac*, in that Letter ſays, I will not: And beſides, the point there mentioned is not the point here in debate; ſo that you have no right to draw any inference from that point to this.

Having now done with every thing neceſſary to the vindication of Sir *Iſaac Newton* and his followers, and thereby driven you entirely out of our intrenchments, I am conſidering whether I ſhould ſally out and attack you in your own. You have thrown up ſome works, I ſee, which at a diſtance make a pretty good appearance, and ſeem capable of defence: But upon taking a nearer view of them, I judge them to be very ſlight and untenable, and to be guarded rather by a new-raiſed, undiſciplined Militia, than any thing of veteran, regular Troops; ſo that it would not be very difficult to carry them by aſſault. But as they ſeem rather deſigned for ſhew, than uſe, more to amuſe yourſelf, than any way to annoy us, I am determined to leave you in poſſeſſion of them.

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* *Analyſt.* p. 27.

Only your ſuppoſition of a * double error in the method of Fluxions, and the uſe you make of it to ſhew how true concluſions are obtained from falſe principles, by means of two contrary miſtakes exactly compenſating one another, has ſomething in it ſo extraordinary, as to require and deſerve a particular conſideration. This darling Phantom, this beloved offspring of your teeming brain, which like *Minerva* iſſuing armed from the head of *Jupiter*, her ſpear in one hand, and her Shield with the Gorgon's head in the other, is to turn all our Mathematicians into ſtocks, and ſtones, and ſtatues, is ſet forth with ſo much art and ſkill, and is dreſſed out in ſo advantageous and pompous a manner, to draw the attention and to dazzle the imagination of the ſpectators, that it would be unpardonable neglect and rudeneſs in me to paſs it by unregarded. I ſhall not therefore content my ſelf with ſaying that one * of theſe errors is already become evaneſcent, i. e. *is nothing, is no error at all*; and that the other of them will likewiſe immediately diſappear like ‡ *the Ghoſt of a departed quantity*, if you exorciſe it with a few words out of the firſt ſection of the *Principia :* On the contrary, I propoſe ſo far to gratify your fondneſs for this hopeful ſcheme, as to give it a fair and full examination.

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* *Analyſt* p. 31 & *ſeq. to* p. 49.

We are to conſider therefore what may be the reaſon, that in the method of Fluxions the concluſions are exactly true: For in the exactneſs of the concluſions we are both agreed; though there be a wide difference between us in reſpect of the means by which Mathematicians arrive at that exactneſs. I conceive that the concluſion is therefore exact, becauſe it is deduced by juſt reaſoning from certain principles. You on the contrary are of opinion that Sir *Iſaac Newton* is guilty of a capital and fundamental error in rejecting the quantity *a b*, ſo often talked of, and * *that the concluſion comes out right, not becauſe the quantity rejected is infinitely ſmall; but becauſe this error is compenſated by another contrary and equal error.* And this you ſay, † *perhaps the Demonſtrator himſelf never knew or thought of.* ‡ *If he had committed only one error, he would not have come at a true ſolution of the Problem. But by virtue of a two-fold miſtake he arrives, though not at ſcience,* yet *at truth. For ſcience it cannot be called, when he proceeds blindfold, and arrives at the truth not knowing how or by what means.* This is the way you account for what you juſtly ſay, *may perhaps ſeem an unaccountable Paradox,* § *that Mathematicians ſhould deduce true Propoſitions from falſe Principles, be right in the concluſion, and yet err in the premiſſes; that error ſhould bring forth truth, though it cannot bring forth ſcience.*

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* *See* p. 46—53. † *Anal.* p. 59.

Now truly, Sir, if this Paradox of yours ſhould be well made out, I muſt confeſs it ought very much to alter the opinion the world has had of Sir *Iſaac Newton*, and occaſion our talking of him in a very different manner from what we have hitherto done. What think you if, inſtead of the greateſt that ever was, we ſhould call him the moſt fortunate, the moſt lucky Mathematician that ever drew a circle? Methinks I ſee the good old Gentleman faſt aſleep and ſnoring in his eaſy-chair, while Dame Fortune is bringing him her apron full of beautiful Theorems and Problems *which he never knows or thinks of*: juſt as the *Athenians* once painted her dragging towns and cities to her favourite General. For what elſe but extreme good fortune could occaſion the concluſions ariſing from his method to be always true and juſt and accurate, when the premiſſes were inaccurate and erroneous and falſe, and only led to right concluſions by means of two errors ever compenſating one another to the utmoſt exactneſs? What luck was here? That when he had made one capital, fundamental, general miſtake, he ſhould happen to make a ſecond, as capital, as fundamental, as general as the firſt; That he ſhould not proceed to commit three or four ſuch miſtakes, but ſtop at the ſecond: That theſe two miſtakes ſhould chance not to lie both the ſame way, but on contrary ſides, ſo that the one might help to correct the other; and laſtly, that the two contrary errors, among all the infinite proportions which they might bear to one another, ſhould happen upon that of a perfect equality; ſo that one might in all poſſible caſes be exactly balanced or compenſated by the other. With a quarter of this good fortune a man might get the 10000 *l.* prize in the preſent Lottery, with a ſingle Ticket.

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* *Anal. p.* 35. † *p.* 36. ‡ *p.* 34. § *p.* 31.

But to come to our point, we are to examine whether the exactneſs of the concluſion is owing to the exact compenſation of of one of theſe errors by the other, or to thoſe errors being utterly inſignificant, being in reality no errors at all. And in order thereto I propoſe to ſee how the concluſion will come out, when only one of theſe errors is committed, ſo that there is nothing to compenſate it.

In your 21 Section, which with its figure I here refer to, the firſt error is ſuppoſed to be the making the ſubtangent or $S = \frac{RB \times PB}{RN}$, inſtead of $S = \frac{RB \times PB.}{RL}$

The ſecond error is making $dy = \frac{p\,dx}{2y}$, inſtead of $dy = \frac{p\,dx - dy\,dy}{2y}$.

If both theſe errors be committed, or if neither of them be committed, the concluſion is agreed to be equally juſt and right, giving $S = 2x$.

If I avoid the firſt of theſe errors, by making $S = \frac{RB \times PB}{L}$; and retain the ſecond, by ſuppoſing $dy = \frac{p\,dx}{2y}$; my concluſion will be $S = 2x \times \frac{2y}{2y + dy}$.

On the other hand if I commit the firſt error, and avoid the ſecond, my concluſion will give me $S = 2x \times \frac{2y + dy}{2y}$.

Now I affirm that theſe two ſeveral values of S, which are the reſult of one error only without any thing to compenſate it, are both true and equally exact with the former value, $2x$, which is the reſult either of two errors, or of none at all. You, I am ſenſible, will diſpute this with me; you will ſay that one of theſe, $2x \times \frac{2y}{2y + dy}$, is leſs than $2x$;

and the latter, $2x \times \frac{2y + dy}{2y}$, is in the ſame proportion bigger than $2x$. I beg leave therefore, for the information of ſome of my readers, to ask you a queſtion. Suppoſing the true ſubtangent $2x$ to be a thouſand miles in length, how much will the ſecond value of that ſubtangent, $2x \times \frac{2y}{2y + dy}$ fall ſhort of a thouſand miles? Will it be a yard, or a foot, or an inch? None of theſe you confeſs, nor the thouſandth, nor the thouſand-millionth part of an inch.

I aſk farther, what then is this difference? Is it poſſible in all the infinity of fractional numbers to find any thing ſmall enough to repreſent it? You own, you confeſs it is not: You muſt confeſs likewiſe, that if theſe three ſeveral values of S were all to be expreſſed in numbers, without being reducible to which, in your * opinion, they can be of no uſe, they muſt every one be expreſſed by 1000, without the leaſt tittle of variation, addition, or diminution. Behold, Gentle Reader, what a mighty ‡ beam here has been diſcovered in the eyes of Mathematicians, in compariſon of which

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* *Analyſt Query* 24.

‡ *Motto to Analyſt.*

all the difficulties in Divinity are but as motes and atoms!

Since therefore theſe errors are wholly inſignificant, my concluſion when reduced to numbers, coming out exactly the ſame, whether the firſt, or ſecond, or neither, or both of theſe errors be committed; and ſince by committing both theſe errors, the calculus, which would otherwiſe, eſpecially in the higher operations, be exceedingly tedious and laborious, is now rendered ſurpriſingly expeditious and eaſy; it ſeems to me that this is ſo far from being any defect in the method of Fluxions, that on the contrary it is one of the greateſt advantages and excellencies of that invention. But you tell me it is not the uſefulneſs of this method that is the matter in diſpute: all the queſtion is whether it be ſcientifical, whether thoſe who uſe it, *ſee their way diſtinctly, or proceed blindfold and arrive at the truth not knowing how or by what means.* I have ſpoken to this before, but muſt add a word or two more in this place. You, Sir, are for avoiding theſe two errors; I am for retaining them. When you avoid them, do not you *ſee your way diſtinctly?* And if I retain them, voluntarily, and with my eyes open;

may I not nevertheleſs clearly ſee the effect of theſe errors, or of either of them, in every ſtep I take and in the concluſion I at laſt come to? May I not therefore likewiſe be ſaid to *ſee my way diſtinctly?* Now, if you and I can ſee our way ſo well, I am afraid it will be conſtrued as great preſumption in us to ſuppoſe that no body does ſo beſides our ſelves: and much more, if we ſhould ſay that the Great Inventor of this method, and the Author of ſo many other wonderful diſcoveries, *never knew or thought of* what to us appears ſo plain and manifeſt; that he who gave us ſo much light, was in the dark himſelf; that he who opened our Eyes, had no ſight of his own. For my part I can never concur with you in thinking that I ſee farther, or go beyond Sir *Iſaac Newton:*

Sed longe ſequor, & veſtigia pronus adoro.

But if you think fit to perſiſt in aſſerting that this affair of a double error is intirely a new diſcovery of your own, which Sir *Iſaac* and his followers *never knew nor thought of,* I have unqueſtionable evidence to convince you of the contrary. I muſt acquaint you therefore with what all his followers

lowers are already appriſed of, that theſe very objections of yours were long ſince foreſeen, and clearly and fully removed by Sir *Iſaac Newton*, in the firſt ſection of the firſt book of his *Principia*; the greater part of which ſection, particularly the firſt and ſeventh *Lemma*, and that admirable *Scholium* at the end of it, was written to this very end and purpoſe only, and to no other in the world.

I have now no more to do, but only to acquit my ſelf of the promiſe I made a while ago, to rectify a miſtake you are fallen into with regard to another of the greateſt men the Engliſh nation has produced. In order to which I muſt here tranſcribe the greater part of the CXXV article of your New Theory of Viſion.

" After reiterated endeavours to appre-
" hend the general Idea of a Triangle, I
" have found it altogether incomprehenſible.
" And ſurely if any one were able to in-
" troduce that Idea into my Mind, it muſt
" be the Author of the *Eſſay concerning Hu-*
" *man Underſtanding*; He, who has ſo far
" diſtinguiſhed himſelf from the generality
" of Writers, by the clearneſs and ſignifi-
" cancy of what he ſays. Let us therefore
" ſee how this celebrated Author deſcribes

“ the general, or abſtract Idea of a Trian-
“ gle. *It muſt be*, ſays he, *neither Oblique*,
“ *nor Rectangular, neither Equilateral, E-*
“ *quicrural, nor Scalenum; but all and none*
“ *of theſe at once. In effect it is ſomewhat*
“ *imperfect that cannot exiſt; an Idea, where-*
“ *in ſome parts of ſeveral different and incon-*
“ *ſiſtent Ideas are put together.* Eſſay on
“ Human Underſtanding. B. iv. C. 7. S. 9.
“ This is the Idea, which he thinks need-
“ ful for the Enlargement of Knowledge,
“ which is the ſubject of Mathematical De-
“ monſtration, and without which we could
“ never come to know any general Propoſi-
“ tion concerning Triangles. That Author
“ acknowledges it doth *require ſome pains*
“ *and skill to form this general Idea of a Tri-*
“ *angle.* Ibid. But had he called to mind
“ what he ſays in another place; to wit,
“ that *Ideas of mixed Modes wherein any in-*
“ *conſiſtent Ideas are put together, cannot ſo*
“ *much as exiſt in the mind*, i. e. be con-
“ ceived. Vid. B. III. C. 10. S. 33. Ibid. I
“ ſay, had this occurred to his Thoughts,
“ it is not improbable he would have own-
“ ed it above all the Pains and Skill he was
“ maſter of, to form the above-mentioned
“ Idea of a Triangle, which is made up

" of manifeſt, ſtaring contradictions. That " a Man who thought ſo much, and laid " ſo great a ſtreſs on clear and determinate " Ideas, ſhould nevertheleſs talk at this rate " ſeems very ſurpriſing.

In this ſection you plainly accuſe Mr. *Locke* of contradicting himſelf in two ſeveral particulars.

1. The above-mentioned Idea of a Triangle, ſay you, is made up of manifeſt, ſtaring contradictions.

2. You repreſent the two following propoſitions of Mr. *Locke* as contradictory one to the other.

It, The general Idea of a Triangle, *is an Idea, wherein ſome parts of ſeveral different and inconſiſtent Ideas are put together.*

Ideas of mixed modes, wherein any inconſiſtent Ideas are put together, cannot ſo much as exiſt in the Mind.

I propoſe to clear up theſe two points, and to ſhew that in neither of them Mr. *Locke* is guilty of contradicting himſelf: but firſt, in order thereto, I muſt take up a little of your time in conſidering the notion of general, or abſtract Ideas. Which pains I am the rather inclined to take becauſe, though I have carefully peruſed what you have written upon this ſubject, I am one of thoſe who ſtill adhere to the vulgar, or rather univerſal error of all Mankind, that neither Geometry, nor any other general ſcience can ſubſiſt without general Ideas.

Though the words abſtract or general Ideas are indifferently uſed by Writers as having the ſame common ſignification; yet as it may be a means of rendering what I have to ſay upon this ſubject ſomething more intelligible, I ſhall beg leave to make a diſtinction between them, not as being different in themſelves, but only in reſpect of the manner in which they are commonly formed or introduced into the mind.

I ſhall confine the name of abſtract Idea to that, which the mind forms to itſelf from the conſideration of ſome number of different ſpecies, by abſtracting from thoſe particular Ideas in which the ſpecies differ from one another, and retaining thoſe in which they agree.

I ſhall call that a general Idea, which may be produced in the mind without any conſideration, or even knowledge, of different Species.

An example will make this very plain. When Mr. *Ray* is forming his Method of

Plants, he obſerves that Mint, and Sage, and Lavender, and Roſemary, and many other Plants, beſides their particular characteriſticks by which they are diſtinguiſhed from one another, have ſome other marks in which they all agree; as in their leaves growing in pairs oppoſite to each other, a monopetalous labiate flower, with four ſeeds growing at the bottom of it, and thoſe incloſed in no other veſſel than the perianthium. By joining together theſe common marks he forms his compound Idea of that Genus of Plants which he calls verticillate: which from his laying aſide, or abſtracting from all the peculiar diſtinguiſhing marks of the ſeveral ſpecies, is properly named an abſtract Idea.

But if Mr. *Ray* will teach me Botany by his Method, he muſt take a different courſe; he muſt begin with me where he himſelf ended; he muſt firſt introduce into my mind the general Idea of a verticillate plant, and afterwards deſcend to particular ſpecies. He tells me a verticillate plant is one whoſe leaves grow in pairs oppoſite to each other, and whoſe flower is monopetalous and labiate, with four ſeeds at the bottom of it, and thoſe incloſed only in the perianthium.

This in me is properly called a general Idea, becauſe I ſhall find it to comprehend all the particular ſpecies of verticillate plants: but I have no reaſon to call it an abſtract Idea, becauſe not knowing as yet any of the particular ſpecies, or their characteriſtick differences, I have nothing to abſtract from.

The abſtract Idea therefore is that of the Maſter or Philoſopher; and the general Idea that of the Diſciple. The former requires, as Mr. *Locke* obſerves, ſome pains and ſkill to form it: the latter demands neither pains nor ſkill, it needs only a little attention to conceive it.

In like manner if a perſon acquainted with the ſeveral ſpecies of Triangles, is from the conſideration of theſe to form an Idea of a Triangle in general; his method will be to examine the ſeveral compound Ideas of the different ſpecies of Triangles, and to diſtinguiſh between ſuch parts of thoſe compound Ideas as are the peculiar characteriſticks of each ſpecies, and ſuch parts as are common to all of them in general. Then connecting theſe laſt together into a new compound Idea, and abſtracting from all the reſt, he will have the abſtract Idea of a Triangle; which is that of a ſpace comprehended by three right lines, add if you pleaſe, containing three angles.

When he has got this Idea himſelf, it is the eaſieſt thing in the world, to give it to another. Let him take a Learner, a Boy, ſuppoſe, who has never learned what a triangle is, much leſs what any particular ſpecies of Triangle is; and tell him a Triangle is a ſpace comprehended by three right lines: I ſay that the Boy, as ſoon as he underſtands the meaning of theſe words, will have acquired the general Idea of a Triangle. If you doubt of it, ſhew him a rectangular Triangle drawn upon paper, and aſk him what it is; he will without heſitation tell you it is a Triangle: afterwards ſhew him ſeparately all the other ſpecies of Triangles, and you will find he knows them every one to be a Triangle. His Idea of a Triangle therefore is general, inaſmuch as it ſuits all the particular ſpecies. And the acquiring this Idea either abſtract, or general, in Teacher or Scholar, ſeems to me to be attended with ſo little difficulty, that I think Mr. *Locke* has ſaid full enough when he declares that the firſt *requires ſome pains and skill to form it*: and it is to me ſurpriſing to hear a Gentleman of your penetration profeſs, that *after reiterated endeavours to apprehend the general Idea of a Triangle, you have found it altogether incomprehenſible.* Put your ſelf but once in the caſe of a Learner, endeavour to diveſt your mind of all your preconceived Geometrical Ideas, and then turn to *Euclid*'s definitions; and I'll venture to aſſure you, you will find no more difficulty in apprehending the general Idea of a Triangle, than in apprehending the Idea of an obliqueangled, or of a ſcalene Triangle, or even that of an Angle alone; there being no objection againſt the firſt, but what may with equal reaſon be brought againſt any of the others; as will eaſily appear to him that conſiders, that an angle in general, an obliqueangled Triangle in general, and a ſcalene Triangle in general can no where exiſt but in Idea only, any more than a Triangle in general.

Having premiſed thus much concerning the abſtract, or general Idea of a Triangle, I come now to examine into your charge againſt Mr. *Locke*, and in the firſt place I muſt take notice that this charge is introduced in an unfair and unjuſt manner. *If any one were able to introduce that Idea into my mind*; ſay you, *it muſt be the Author of the*

the Eſſay concerning Human Underſtanding; &c. *Let us therefore ſee how this celebrated Author deſcribes the general, or abſtract Idea of a Triangle.* Would not any body imagine from theſe words that Mr. *Locke* were here purpoſely *deſcribing* this Idea, in order to *introduce it into the mind* of one who had it not already? If that were his intention, it is certainly a moſt miſerable deſcription; ſince no perſon living who does not already know what a Triangle is, can ever have *that Idea introduced into his mind* from what Mr. *Locke* here lays down. And yet that Idea is introduced into the mind with all the eaſe in the world by what he gives us to underſtand in another * place, that the Idea of a Triangle is that of *three lines, including a ſpace.* Could he poſſibly talk ſo clearly in one place, and ſo cloudily in another, if his intention were the ſame in both? Is it not plain to any one who attentively reads the paſſage you refer to, that his intention there was not to *deſcribe* the general Idea of a Triangle, but only to ſhew from the ſeeming inconſiſtencies in that Idea, ſuppoſed to be already known, that it required ſome pains and ſkill to form it, as well as other abſtract Ideas? Obſerve his words, "For "abſtract Ideas are not ſo obvious or eaſy "to children, or the yet unexerciſed mind, "as particular ones. If they ſeem ſo to "grown men, 'tis only becauſe by conſtant "and familiar uſe they are made ſo. For "when we nicely reflect upon them, we "ſhall find, that general Ideas are fictions "and contrivances of the mind, that carry "difficulty with them, and do not ſo eaſily "offer themſelves, as we are apt to ima-"gine. For example, Does it not require "ſome pains and ſkill to form the general "Idea of a Triangle? (Which yet is none "of the moſt abſtract, comprehenſive and "difficult.) For it muſt be neither oblique, "nor rectangle, neither equilateral, equicru-"ral, nor ſcalenon; but all and none of "theſe at once. In effect, it is ſomething "imperfect, that cannot exiſt; an Idea where-"in ſome parts of ſeveral different and in-"conſiſtent Ideas are put together."

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* *Eſſay on Hum. Underſt. B. II. C.* 31. *S.* 6.

We come now to the manifeſt, ſtaring contradictions, contained in this Idea of a Triangle: the firſt of which, I ſuppoſe, is contained in theſe words, *all and none of theſe at once.* The *Enantioſis*, I confeſs, is pretty ſtrong: and yet the meaning of it is plainly no more than this, that the general Idea of a Triangle is a part of the Idea of every ſpecies of Triangles here enumerated, but is not the intire Idea of any one of them; is common to them all, and confined to none. *It is ſomething imperfect that cannot exiſt*, may poſſibly be another of your contradictions. It does not appear ſo to me. For every individual Triangle, every Triangle that can exiſt, muſt be ſomething more than a ſpace included by three lines, it muſt alſo have the characteriſtick mark of ſome one of the particular ſpecies of Triangles; without which it would be *imperfect*, *it could not exiſt*, which is what Mr. *Locke* here ſays of a Triangle in general.

2. But the great contradiction of all ſeems to lie in the two following propoſitions, which are brought together from different parts of Mr. *Locke*'s works, and ſet to ſtare one another in the face to diſgrace their Author.

It is an Idea, wherein ſome parts of ſeveral different and inconſiſtent Ideas are put together.

Ideas of mixed modes, wherein any inconſiſtent Ideas are put together, cannot ſo much as exiſt in the mind.

Here, Sir, I ſtrongly apprehend you are fallen into one of thoſe traps, which this Great Man would ſometimes divert himſelf with ſetting to catch unwary cavillers, the *Homines ſtolidos & ad depugnandum paratos*, that I mentioned a while ago. Had his firſt propoſition run thus, It is an Idea, wherein ſeveral different and inconſiſtent Ideas are put together, it would undoubtedly have been contradictory to the ſecond. But that is not the caſe: pray obſerve the words of this cautious and accurate Writer. It is an Idea, wherein SOME PARTS OF ſeveral different and inconſiſtent Ideas are put together. Now, we know that the ſeveral compound Ideas of a rectangled, an oblique, and an acuteangled Triangle are different and inconſiſtent one with another. No two of them can be put together ſo as jointly to exiſt or be conceived in the mind. Likewiſe the ſeveral compound Ideas of an equilateral, equicrural, and ſcalene triangle are inconſiſtent with one another. But yet ſome parts of one of theſe inconſiſtent Ideas are not only conſiſtent, but are perfectly the ſame with ſome parts of another. To ſhew this I beg leave to divide two of theſe inconſiſtent Ideas into ſeveral parts.

The compound Idea of a rectangled Triangle may be divided into theſe parts.	The compound Idea of an acuteangled Triangle may be divided into theſe parts.
1. A plain ſpace,	1. A plain ſpace,
2. Comprehended by right lines,	2. Comprehended by right lines,
3. Three in number,	3. Three in number,
4. Containing three angles,	4. Containing three angles,
5. One right, two acute.	5. All acute.

There is, we ſee, no difference between the four firſt parts of the compound Idea of a rectangled Triangle, and the four firſt parts of that of an acuteangled Triangle: it is owing to the fifth part alone of each Idea, that theſe two Ideas are different and inconſiſtent. And as it is eaſy to ſee that theſe four firſt parts are the ſame in all the other particular Species of Triangles; and that the ſame four parts do compoſe the general Idea of a Triangle; it is plain that the general Idea of a Triangle is an Idea, wherein SOME PARTS OF ſeveral different and inconſiſtent Ideas are put together.

The firſt therefore of the two propoſitions in queſtion is undoubtedly true; and as theſe parts are no way inconſiſtent with one another; it is manifeſt that the ſecond propoſition is not contradictory, or at all repugnant to the firſt.

I come now, Sir, to take my leave of you, and hope that if an honeſt zeal for truth in the firſt place, and in the ſecond for the reputation of thoſe Gentlemen to whom I conceive the whole body of mankind, at leaſt I muſt acknowledge my ſelf to be highly indebted, has given occaſion not only of differing from you, but even of reprehending you with the utmoſt freedom wherever I thought the truth and your behaviour required it; you will not impute the liberty I have taken to any diſreſpect for your perſon, which I am an utter ſtranger to, though I have a very great eſteem and value for your uncommon abilities and many of your writings, and am with ſincere reſpect,

SIR,
Your moſt obedient,
Humble Servant,

PHILALETHES CANTABRIGIENSIS.

FINIS.

Keynes nos. 32, 35, 36, 38